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A STRONGER CHARACTERIZATION OF DECLINING RISK AVERSION

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1. THE ARROW–PRATT AND ROSS CHARACTERIZATIONS

THE HYPOTHESIS that absolute[2] risk aversion is a nonincreasing function of initial or "base" wealth is one of the fundamental assumptions of the theory of individual behavior toward risk, and has been adopted or suggested by virtually every researcher who has addressed the question of wealth and risk aversion (e.g. Arrow [**1**], Hicks [**6**], Pratt [**13**], Raiffa [**14**], Yaari [**20**]). The standard behavioral characterizations of declining risk aversion, due to Arrow [**1**] and Pratt [**13**], are:

(A.1) If $x, \Delta x \geq 0$, $E[\tilde{\epsilon}] = 0$, and π_0, π_1 are such that $x - \pi_0 \sim x + \tilde{\epsilon}$ and $x + \Delta x - \pi_1 \sim x + \Delta x + \tilde{\epsilon}$, then $\pi_0 \geq \pi_1$, and

(A.2) if $x, \Delta x, r \geq 0$, $E[\tilde{\epsilon}] = 0$, $\tilde{z} = x + r + \tilde{\epsilon}$, and α_0, α_1 yield the most preferred distributions of the form $(1-\alpha)x + \alpha\tilde{z}$ and $(1-\alpha)x + \Delta x + \alpha\tilde{z}$ respectively, then $\alpha_0 \leq \alpha_1$,

where "$\tilde{\ }$" denotes a stochastic variable, "$\sim$" denotes indifference between two random wealth distributions, and $E[\cdot]$ denotes statistical expectation. The well known interpretations of these conditions are (A.1) that the premium π that the individual would be just willing to pay for complete insurance is a nonincreasing function of base wealth, and (A.2) that in allocating a quantity of investible funds between a riskless asset and a risky asset with higher mean return, the absolute demand for the risky asset is nondecreasing in the level of investible funds. Arrow [**1**] and Pratt [**13**] have shown that if the individual is an expected utility maximizer with twice differentiable concave utility function $U(\cdot)$, then these two behavioral conditions are equivalent to the mathematical condition:

(A.3) $-U''(x)/U'(x)$ is nonincreasing in x.

The Arrow–Pratt characterization (A.1)–(A.3) has been widely adopted in the literature. However, as Yaari has noted, the general notion that "greater wealth can never lead to greater risk aversion . . . is not really an axiom, but a heading for a whole class of possible axioms which, while similar in spirit, may differ greatly" [**20**, p. 320].[3] In particular, Ross [**15**] has recently called attention to the fact that the real world seldom offers the *complete* certainty that the above concepts of "complete insurance" and "risk free asset" presume. Individual insurance contracts typically only cover losses due to one or more specific causes (fire, theft, etc.), and while "complete" insurance via some combination of policies may be possible in principle, it is typically not undertaken by individuals. Similarly, once there is any uncertainty about future price levels or future

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[2] Throughout this paper we are concerned solely with the relationship of absolute risk aversion to wealth, and not with the less widely maintained hypothesis of increasing *relative* risk aversion (see Arrow [**1**, pp. 94–109] and Stiglitz [**18**]).

[3] See, for example, Yaari [**20**], Mayshar [**11**], and Zubiri [**21**] for alternative approaches to the asset demand condition (A.2).

relative prices, no nominal or real asset can be completely risk free. It is thus clear that if any characterization of declining risk aversion is to be relevant, it must be strong enough to imply "decreasingly risk averse" behavior in a world of partial insurance, uninsurable risks, and no riskless asset.

Ross has shown that, for the general class of actuarially neutral risks,[4] the Arrow–Pratt characterization does not meet this requirement. For example, it is possible for an individual to satisfy (A.3), yet still be willing to pay a higher premium for insurance against a given risk $\tilde{\epsilon}$ as the result of a constant increment Δx to a stochastic base wealth $\tilde{x}$. Similarly, if such an individual is allocating a fixed amount of funds between two risky assets, one with a higher mean and greater risk than the other, a constant increment to ex post wealth may lead to a reallocation of the original investment funds in favor of the less risky asset. In light of this, Ross has offered the following pair of stronger behavioral conditions:

(B.1) If $\tilde{x}, \Delta x \geq 0$, $E[\tilde{\epsilon} \mid x] = 0$ for all x, and π_0, π_1 are such that $\tilde{x} - \pi_0 \sim \tilde{x} + \tilde{\epsilon}$ and $\tilde{x} + \Delta x - \pi_1 \sim \tilde{x} + \Delta x + \tilde{\epsilon}$, then $\pi_0 \geq \pi_1$, and

(B.2) if $\tilde{x}, \Delta x \geq 0$, $E[\tilde{z} \mid x] \geq x$ for all x, and α_0, α_1 yield the most preferred distributions of the form $(1 - \alpha)\tilde{x} + \alpha\tilde{z}$ and $(1 - \alpha)\tilde{x} + \Delta x + \alpha\tilde{z}$ respectively, then $\alpha_0 \leq \alpha_1$,

and has shown that they are exhibited by any risk averse expected utility maximizer satisfying:

(B.3) $U'''(x_a)/U''(x_a) \leq U''(x_b)/U'(x_b)$ for all x_a, x_b (or equivalently, that $-U''(x + c)/U'(x)$ is nonincreasing in x for all c).[5]

While this reformulation is clearly a step in the direction of realism, it still possesses a particularly unrealistic aspect, namely the requirement that the increment Δx to base wealth be nonstochastic. Consider the following, for example:

(a) If sources of income or wealth are in general stochastic, then any increment in wealth due to the acquisition of a new or additional such source (e.g. inheritance of a stock portfolio) will be similarly stochastic.

(b) In the case of a single source of wealth, say the ownership of a firm's profit stream, the exact increment due to some beneficial change (e.g. a shift in preferences towards the firm's product) will often depend on the values of other random variables in the economy (aggregate income, prices of substitutes, etc.).

(c) Finally, even if the pre-tax increment to a stochastic initial wealth were a constant, if the tax schedule is nonlinear, then the post-tax increment will be stochastic.

Since it will be shown that condition (B.3) is insufficient to imply decreasingly risk averse behavior when wealth increments are stochastic, it follows that the Ross character-

[4] In the special case where the risk $\tilde{\epsilon}$ is exactly stochastically independent of a random initial wealth $\tilde{x}$, Kihlstrom, Romer, and Williams [**8**] have shown that (A.3) is sufficient to ensure that a constant increment Δx will lower the risk premium π. However, since the work of Rothschild and Stiglitz [**16**, **17**] it has been clear that the class of independent additive risks is but a small subset of the class of general mean preserving increases in risk, which are characterized by the addition of any $\tilde{\epsilon}$ satisfying $E[\tilde{\epsilon} \mid x] = 0$ for all x. See McFadden [**12**] for an argument why individuals might tend not to view additional risks and base wealth as independent (or uncorrelated), as well as the related work of Hildreth [**7**] and Levy and Kroll [**9**].

[5] Ross [**15**] also gives a related strengthening of the Arrow–Pratt characterization of comparative risk aversion between individuals (see also Kihlstrom, Romer, and Williams [**8**]).

ization is still too specialized (i.e. applies to too small a class of situations) to have any effective predictive power. Similarly, it is hard to think of any positive or normative arguments for the decreasingly risk averse behavior described in (B.1) and (B.2) which would not also suggest declining risk aversion in cases where the distribution of base wealth undergoes a general rightward shift, as opposed to an exact horizontal translation. Indeed, the maxim that economic models should be robust to the general uncertain nature of *all* economic variables ought to apply first and foremost to those which explicitly purport to describe behavior toward risk.

The purpose of this note is to characterize the phenomenon of declining risk aversion in a world where neither initial wealth nor wealth increments are necessarily nonstochastic, by considering the natural extensions of the above risk premium and asset demand conditions to the case of a stochastic nonnegative wealth increment $\widetilde{\Delta x}$. Surprisingly, it is found that unless the individual happens to be globally risk neutral (i.e. risk aversion is constant at zero), the hypothesis of expected utility maximization is too restrictive to allow the individual to exhibit nonincreasing risk aversion in this more general (and presumably more realistic) setting, although such behavior may be exhibited by individuals with more general preferences over random wealth distributions. Besides offering a simple mathematical characterization of preferences which exhibit decreasingly risk averse behavior in this more general sense, we show that under suitable regularity conditions the generalized insurance premium and generalized asset demand conditions continue to be equivalent characterizations of behavior.

2. "GENERALIZED NONINCREASING RISK AVERSION"

In light of the above discussion, we adopt the following behavioral characterizations:

(C.1) If $\tilde{x}, \widetilde{\Delta x} \geq 0$, $E[\tilde{\epsilon} \mid x] = E[\tilde{\epsilon} \mid x + \Delta x] = 0$ for all $x, x + \Delta x$, and π_0, π_1 are such that $\tilde{x} - \pi_0 \sim \tilde{x} + \tilde{\epsilon}$ and $\tilde{x} + \widetilde{\Delta x} - \pi_1 \sim \tilde{x} + \widetilde{\Delta x} + \tilde{\epsilon}$, then $\pi_0 \geq \pi_1$, and

(C.2) if $\tilde{x}, \widetilde{\Delta x}, r \geq 0$, $E[\tilde{\epsilon} \mid x] = E[\tilde{\epsilon} \mid x + \Delta x] = 0$ for all $x, x + \Delta x$, $\tilde{z} = \tilde{x} + r + \tilde{\epsilon}$, and α_0, α_1 yield the most preferred distributions of the form $(1 - \alpha)\tilde{x} + \alpha\tilde{z}$ and $(1 - \alpha)\tilde{x} + \widetilde{\Delta x} + \alpha\tilde{z}$ respectively, then $\alpha_0 \leq \alpha_1$.[6,7]

Two aspects of these conditions are worth noting. First, for reasons discussed in footnote 4, we adopt the new condition $E[\tilde{\epsilon} \mid x + \Delta x] = 0$ so that the addition of $\tilde{\epsilon}$ represents an increase in risk with respect to both the original and the new distributions of base wealth (i.e. both $\tilde{x}$ and $\tilde{x} + \widetilde{\Delta x}$). Second, although (C.2) formally represents $\widetilde{\Delta x}$ as an increment to ex post wealth (e.g. the acquisition of a nonsaleable asset), it includes as a special case increments in the level of investible funds. To see this, note that a proportionate increase t in the level of investment funds will increase the absolute demand for the riskier asset if

[6] Note that the present characterization of $\tilde{z}$ as "riskier and with higher mean return than $\tilde{x}$" is not the same as used by Ross (condition (B.2) and [**15**, pp. 631–33, 637–38]), who (in our notation) requires merely that $E[\tilde{z} \mid x] \geq x$ for all x. However, this latter condition is not sufficient to ensure that $\tilde{z}$ is in any sense riskier than, or even as risky as, $\tilde{x}$ (e.g. let $\tilde{z}$ take on a constant value to the right of the support of $\tilde{x}$).

[7] Although this condition assumes no risk free asset, it does assume that the individual has only two assets to choose among. Cass and Stiglitz [**2**] and Hart [**5**] have shown that with a risk free asset and two or more risky assets, the Arrow–Pratt condition (A.3) is not sufficient to ensure that the total demand for risky assets increases with wealth. Because we are concerned here with an essentially unrelated aspect of the notation of declining risk aversion, we do not consider the case of one riskless and two or more risky assets.

and only if the value of α which yields the most preferred distribution of the form $(1-\alpha)\tilde{x}+t\tilde{x}+\alpha\tilde{z}$ is greater than α_0, which since $t\tilde{x}$ is nonnegative and $E[\tilde{\epsilon}\,|\,x+tx]\equiv 0$ if and only if $E[\tilde{\epsilon}\,|\,x]\equiv 0$, is a special case of (C.2). By contrast (and continuing to assume no riskless asset), the case of an increase in investible funds is evidently not a special case of the Ross condition (B.2).[8]

In modelling this more general type of behavior we retain the standard choice theoretic assumption that the individual chooses among alternative wealth distributions as if maximizing a preference functional $V(F)$ defined over the set $D[0,M]$ of all cumulative distribution functions $F(\cdot)$ over $[0,M]$, where $M<\infty$ may be arbitrarily large.[9] If the individual's preferences satisfy the axioms of expected utility theory, we can represent the preference functional as having the form $V(F)\equiv\int U(x)\,dF(x)$,[10] where $U(\cdot)$ is the von Neumann–Morgenstern utility of wealth function. In other words, $V(\cdot)$ can be represented as a linear functional over $D[0,M]$, or as commonly phrased, as "linear in the probabilities."

Although the expected utility axioms are often regarded as normatively appealing, unless the utility function $U(\cdot)$ is itself linear (i.e. risk aversion is identically zero), the expected utility hypothesis is incompatible with the (perhaps equally appealing) conditions (C.1) and (C.2). Although this will follow formally from Theorem 1, the intuition may be gleaned from a simple example. Let the triple $(\tilde{x},\widetilde{\Delta x},\tilde{\epsilon})$ take on the values $(x^*,d,0)$, $(x^*,0,e)$, and $(x^*,0,-e)$, each with probability $1/3$, where $x^*,d,e>0$, so that this joint distribution satisfies the conditions of condition (C.1). Thus the risk premia π_0 and π_1 will solve:

$$U(x^*-\pi_0)=\tfrac{1}{3}[U(x^*+e)+U(x^*)+U(x^*-e)] \tag{1}$$

and

$$\tfrac{1}{3}U(x^*+d-\pi_1)+\tfrac{2}{3}U(x^*-\pi_1)=\tfrac{1}{3}[U(x^*+d)+U(x^*+e)+U(x^*-e)] \tag{2}$$

so that

$$\begin{aligned}&[U(x^*+d)-U(x^*+d-\pi_1)]-[U(x^*)-U(x^*-\pi_1)]\\&\quad=3[U(x^*-\pi_1)-U(x^*-\pi_0)].\end{aligned} \tag{3}$$

In this case, if the utility function is increasing and strictly concave in the relevant neighborhood, the left side of (3) will be negative, which implies $\pi_1>\pi_0$. Similarly, if $U(\cdot)$ is increasing and strictly convex, π_0 and π_1 will be negative, so we again have $\pi_1>\pi_0$. Since the magnitudes of d and e may be arbitrarily small, it follows that unless $U(\cdot)$ is linear in the relevant neighborhood (so that $\pi_0=\pi_1=0$), an expected utility maximizer cannot even satisfy (C.1) "in the small."

The intuition behind this incompatibility is straightforward. Recall that the risk premium π is such that the loss (gain) in expected utility from paying (receiving) the premium exactly equals the loss (gain) in expected utility from bearing the risk $\tilde{\epsilon}$. However, in the case of concavity, to the extent that the increment in wealth occurs in states of nature different from those in which the risk occurs (as in the example), it will serve to drive down the (expected) marginal utility of wealth while leaving the loss in expected utility due to the risk unchanged, thus increasing the size of the premium the individual would pay to avoid the risk. In the case of convexity, wealth increments which occur in states other than where the risk occurs serve to drive up the expected marginal utility of wealth, which serves to lower the compensation the individual requires for giving

[8] The Ross characterization of increasing (or decreasing) *relative* risk aversion, however, does include the case of an increment in the level of investible funds ([**15**, p. 638]).

[9] For simplicity, we assume M large enough so that the supports of all relevant random variables ($\tilde{x}$, $\tilde{x}+\widetilde{\Delta x}+\tilde{\epsilon}$, etc.) lie in $[0,M]$.

[10] Throughout this paper all integrals will be over $[0,M]$ unless otherwise specified.

up the risk (i.e. raises the value of π upwards toward zero). In other words, since expected utility implies that preferences are separable in the state-contingent payoffs, an increase in wealth in one state cannot in general affect attitudes toward risk in other states in a manner sufficient to ensure generalized nonincreasing risk aversion, even when the wealth increment $\widetilde{\Delta x}$ and the insurable risk $\tilde{\epsilon}$ are "small."

It might seem that abandoning the expected utility model is a high analytic price to pay to achieve this more general characterization of declining risk aversion. However, in **[10]** it was demonstrated that this need not be the case, and that it is indeed possible to drop the assumption that $V(\cdot)$ is linear and, retaining only the condition that $V(\cdot)$ be a *differentiable* function of $F(\cdot)$ (i.e. that preferences are "smooth"), directly adapt the main body of expected utility analysis to individuals who do not necessarily satisfy the expected utility hypothesis.

Specifically, we adopt the topology of weak convergence over the choice set $D[0, M]$ and assume that the preference functional $V(\cdot)$ is (once) Fréchet differentiable with respect to $F(\cdot)$. In **[10]** this was shown to imply that there exists at each distribution $F_0(\cdot)$ in $D[0, M]$ a "local utility function" $U(\cdot\,; F_0)$ over $[0, M]$ such that for any other distribution $F^*(\cdot)$ in $D[0, M]$,

$$V(F^*) - V(F_0) = \int U(\omega; F_0)[dF^*(\omega) - dF_0(\omega)] + o(\|F^* - F_0\|), \tag{4}$$

where $o(\cdot)$ denotes a function of higher order than its argument, and $\|\cdot\|$ is the standard L^1 norm. Thus, just as with differentiable functions over R^n, the difference $V(F^*) - V(F_0)$ is seen to consist of a first order (i.e. linear) term in $F^*(\cdot) - F_0(\cdot)$ plus a higher order term, with the linear term representable as the difference in the expectations of the local utility function $U(\cdot\,; F_0)$ with respect to the distributions $F^*(\cdot)$ and $F_0(\cdot)$.

It is also clear that, just as in standard multivariate calculus, many of the local properties of the preference functional $V(\cdot)$ about a given distribution $F_0(\cdot)$ are determined by the properties of the local utility function $U(\cdot\,; F_0)$, so that, for example, if $U(x; F_0)$ is concave in x, the individual will be averse to all small mean preserving increases in risk about the initial distribution $F_0(\cdot)$. It was also shown in **[10]** that such relationships often had global extensions: for example, a necessary and sufficient condition for a Fréchet differentiable preference functional $V(\cdot)$ to be averse to all (small or large) mean preserving increases in risk is that the local utility functions $U(x; F)$ be concave in x for all $F(\cdot)$ in $D[0, M]$.

Following Arrow–Pratt and Ross, we assume that the first and second derivatives $U_1(x; F)$ and $U_{11}(x; F)$ of the local utility functions exist and are continuous in x, with $U_1(x; F)$ everywhere positive.[11] In addition, because the study of other than "regular" optima in the asset demand case is beyond the scope of this paper, we adopt the following condition, a generalization of the condition that indifference curves in the (σ, μ) plane are upward sloping and bowed downwards, which serves to rule out both risk lovers (who would always choose $\alpha = \infty$) as well as "plungers" as in the classic study of Tobin **[19]**:

DEFINITION: A risk averse individual is said to be a *diversifier*[12] if, for all $\tilde{x}, \tilde{z} \geq 0$ such that $\tilde{z} = \tilde{x} + r + \tilde{\epsilon}$ with $r > 0$ and $E[\tilde{\epsilon} \mid x] = 0$ for all x, the individual's preferences over the set of random wealths $\{(1-\alpha)\tilde{x} + \alpha\tilde{z}\}_\alpha$ are strictly quasiconcave in α.[13]

[11] In **[10]** the condition $U_1(\cdot\,; F) \geq 0$ was shown, as in expected utility theory, to be a necessary and sufficient condition for $V(\cdot)$ to exhibit "monotonicity," i.e. weak preference for first order stochastically dominating distributions.

[12] Because of the different contexts, this differs from the definition of "diversifier" used in **[10]**.

[13] This condition ensures that preferences are either strictly monotonic in α or else that there is a unique optimal α and that preferences are strictly increasing in α below this value and strictly decreasing above it. Note that this condition implies that if $(\tilde{x}, \widetilde{\Delta x}, \tilde{z})$ satisfy the conditions of condition (C.2) then preferences over the set $\{(1-\alpha)\tilde{x} + \widetilde{\Delta x} + \alpha\tilde{z}\}_\alpha$ will also be strictly quasiconcave in α.

Although we do not restrict short sales of either asset, it is clear that the optimal α for any diversifier will be nonnegative. We thus have:

THEOREM 1: *The following two properties of a Fréchet differentiable preference functional $V(\cdot)$ over $D[0, M]$ with twice continuously differentiable local utility functions $U(\cdot; F)$ with $U_1(\cdot; F) > 0$ are equivalent:*

(i) *the term $-U_{11}(y; F)/\int U_1(\omega; F)\,dF(\omega)$ is everywhere nonincreasing in y and $F(\cdot)$ (i.e. $-U_{11}(y^*; F^*)/\int U_1(\omega; F^*)\,dF^*(\omega) \leq -U_{11}(y; F)/\int U_1(\omega; F)\,dF(\omega)$ whenever $y^* \geq y$ and $F^*(\cdot)$ equals or stochastically dominates*[14] *$F(\cdot)$); and*

(ii) *the insurance premium condition* (C.1).

If the individual is a diversifier, these conditions are in turn equivalent to:

(iii) *the asset demand condition* (C.2). (*Proof in Appendix.*)

Condition (i) of the above theorem has a straightforward interpretation. Starting from an initial distribution $F(\cdot)$ we have that, just as in expected utility theory, the term $-\frac{1}{2}U_{11}(y; F)$ gives the effect of an infinitesimal increase in risk about the outcome value y on the expectation of $U(\cdot; F)$, and hence, by (4), on $V(\cdot)$. Similarly (and again starting from $F(\cdot)$), the term $-\int U_1(\omega; F)\,dF(\omega)$ gives the effect on $V(\cdot)$ of a differential premium payment. The ratio $-\frac{1}{2}U_{11}(y; F)/\int U_1(\omega; F)\,dF(\omega)$ thus gives the value of the premium that individual would be just willing to pay to avoid a differential increase in risk about the outcome value y.

Now, if the individual is to exhibit generalized nonincreasing risk aversion, the shift in the distribution of base wealth from $F_{\tilde{x}}(\cdot)$ to $F_{\tilde{x}+\widetilde{\Delta x}}(\cdot)$ (using obvious notation) must not increase the value of this premium. If the risk we are considering occurs in states of nature where $\Delta x = 0$, it will continue to occur about the outcome value y, so that the stochastically dominating shift in $F(\cdot)$ from $F_{\tilde{x}}(\cdot)$ to $F_{\tilde{x}+\widetilde{\Delta x}}(\cdot)$ should not increase the value of the term $-U_{11}(y; F)/\int U_1(\omega; F)\,dF(\omega)$. If the risk occurs in states where $\Delta x > 0$, the outcome value y about which it occurs will also increase, so that increases in y should also not increase the value of $-U_{11}(y; F)/\int U_1(\omega; F)\,dF(\omega)$. Since it is possible for either of these effects to outweigh the other, we therefore require that $-U_{11}(y; F)/\int U_1(\omega; F)\,dF(\omega)$ be nonincreasing in both y and $F(\cdot)$. Finally, note that while this heuristic argument applies only to the case of a differential increase in risk, it turns out, as with the analysis of Pratt [**13**], that condition (i) is necessary and sufficient for generalized nonincreasing risk aversion "in the large" as well.

In [**10**] a simple class of nonlinear preference functionals over probability distributions was offered, namely those of the form

$$\bar{V}(F) \equiv \int R(\omega)\,dF(\omega) \pm \frac{1}{2}\left[\int S(\omega)\,dF(\omega)\right]^2, \tag{5}$$

which are seen to be "quadratic in the probabilities," and with local utility functions

$$\bar{U}(x; F) = R(x) \pm S(x) \cdot \int S(\omega)\,dF(\omega). \tag{6}$$

A simple example of this functional form is

$$\bar{\bar{V}}(F) \equiv \int \omega\,dF(\omega) - \frac{1}{2}\left[\int e^{-\omega}\,dF(\omega)\right]^2, \tag{7}$$

[14] Throughout this paper, "stochastic dominance" refers to first order stochastic dominance (see Hadar and Russell [**3**]). It is clear that the "nonincreasing in $F(\cdot)$" part of this condition cannot be satisfied by an expected utility maximizer for whom $U''(\cdot)$ is anywhere nonzero.

with local utility function

$$\overline{\overline{U}}(x;F) = x - e^{-x}\cdot\int e^{-\omega}\,dF(\omega). \tag{8}$$

Since $\overline{\overline{U}}(x;F)$ is seen to be increasing and concave in x, it follows from Theorems 1 and 2 of **[10]** that $\overline{\overline{V}}(\cdot)$ exhibits both monotonicity (see footnote 11) and global risk aversion. In addition, since the term

$$-\overline{\overline{U}}_{11}(y;F)\Big/\int\overline{\overline{U}}_1(\omega;F)\,dF(\omega) = \frac{e^{-y}\cdot\int e^{-\omega}\,dF(\omega)}{1+\left[\int e^{-\omega}\,dF(\omega)\right]^2} \tag{9}$$

is seen to be decreasing in both y and $F(\cdot)$,[15] and since an individual with this preference functional may be shown to be a diversifier,[16] it follows that $\overline{\overline{V}}(\cdot)$ satisfies both conditions (C.1) and (C.2) as well.

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APPENDIX

NOTATION: In the following, $F_{\tilde{x}+\tilde{\epsilon}}(\cdot)$, $F_{\tilde{x},\tilde{z}}(\cdot,\cdot)$ denote the cumulative distribution functions of $\tilde{x}+\tilde{\epsilon}$, $(\tilde{x},\tilde{z})$, etc., and $G_c(\cdot)$, $G_{c,d}(\cdot,\cdot)$ the distributions which assign unit probability to the points c, (c,d), etc. (See also footnotes 9, 10, 11, 13, and 14.)

PROOF OF THEOREM 1: (i)$\rightarrow$(ii): Given the joint distribution of $(\tilde{x},\widetilde{\Delta x},\tilde{\epsilon})$, let $\pi(\alpha)$ solve $V(F_{\tilde{x}+\tilde{\epsilon}}) = V(F_{\tilde{x}-\pi_0}) = V((1-\alpha)F_{\tilde{x}+\tilde{\epsilon}-\pi(\alpha)} + \alpha F_{\tilde{x}-\pi(\alpha)})$ and let $F(\cdot,\alpha) = (1-\alpha)F_{\tilde{x}+\tilde{\epsilon}-\pi(\alpha)}(\cdot) + \alpha F_{\tilde{x}-\pi(\alpha)}(\cdot)$ for $\alpha\in[0,1]$, so that $\pi(0)=0$, $\pi(1)=\pi_0$, $F(\cdot,0)=F_{\tilde{x}+\tilde{\epsilon}}(\cdot)$, and $F(\cdot,1)=F_{\tilde{x}-\pi_0}(\cdot)$. From equation (4), we have that for all $\bar{\alpha}\in[0,1]$

$$0 = \left.\frac{dV(F(\cdot,\alpha))}{d\alpha}\right|_{\bar{\alpha}} = -\int U(\omega;F(\cdot,\bar{\alpha}))\left[dF_{\tilde{x}+\tilde{\epsilon}-\pi(\bar{\alpha})}(\omega) - dF_{\tilde{x}-\pi(\bar{\alpha})}(\omega)\right]$$
$$-\pi'(\bar{\alpha})\cdot\int U_1(\omega;F(\cdot,\bar{\alpha}))\left[(1-\bar{\alpha})\,dF_{\tilde{x}+\tilde{\epsilon}-\pi(\bar{\alpha})}(\omega) + \bar{\alpha}\,dF_{\tilde{x}-\pi(\bar{\alpha})}(\omega)\right],$$

so that

$$\pi'(\bar{\alpha}) = \frac{-\int U(\omega;F(\cdot,\bar{\alpha}))\left[dF_{\tilde{x}+\tilde{\epsilon}-\pi(\bar{\alpha})}(\omega) - dF_{\tilde{x}-\pi(\alpha)}(\omega)\right]}{\int U_1(\omega;F(\cdot,\bar{\alpha}))\left[(1-\bar{\alpha})\,dF_{\tilde{x}+\tilde{\epsilon}-\pi(\bar{\alpha})}(\omega) + \bar{\alpha}\,dF_{\tilde{x}-\pi(\bar{\alpha})}(\omega)\right]}. \tag{10}$$

Defining $\bar{F}(\cdot,\alpha) \equiv (1-\alpha)F_{\tilde{x}+\widetilde{\Delta x}+\tilde{\epsilon}-\pi(\alpha)}(\cdot) + \alpha F_{\tilde{x}+\widetilde{\Delta x}-\pi(\alpha)}(\cdot)$ for $\alpha\in[0,1]$ yields

$$\left.\frac{dV(\bar{F}(\cdot,\alpha))}{d\alpha}\right|_{\bar{\alpha}} = -\int U(\omega;\bar{F}(\cdot,\bar{\alpha}))\left[dF_{\tilde{x}+\widetilde{\Delta x}+\tilde{\epsilon}-\pi(\bar{\alpha})}(\omega) - dF_{\tilde{x}+\widetilde{\Delta x}-\pi(\bar{\alpha})}(\omega)\right]$$
$$-\pi'(\bar{\alpha})\cdot\int U_1(\omega;\bar{F}(\cdot,\bar{\alpha}))\Big[(1-\bar{\alpha})\,dF_{\tilde{x}+\widetilde{\Delta x}+\tilde{\epsilon}-\pi(\bar{\alpha})}(\omega)$$
$$+\bar{\alpha}\,dF_{\tilde{x}+\widetilde{\Delta x}-\pi(\bar{\alpha})}(\omega)\Big],$$

[15] The right side of (9) is clearly decreasing in y. It is also decreasing in $F(\cdot)$ since $r/(1+r^2)$ is increasing in r over [0, 1] and $\int e^{-\omega}\,dF(\omega)\in[0,1]$ is decreasing in $F(\cdot)$.

[16] It is straightforward to verify that $d^2\overline{\overline{V}}(F_{(1-\alpha)\tilde{x}+\alpha\tilde{z}})/d\alpha^2 < 0$ whenever $\tilde{z}-\tilde{x}$ is not identically zero.

which, since $U_1(\omega; F) > 0$, has the same sign as

$$(11)\qquad \frac{-\int U(\omega;\bar{F}(\cdot,\bar{\alpha}))[dF_{\tilde{x}+\widetilde{\Delta x}+\tilde{\epsilon}-\pi(\bar{\alpha})}(\omega)-dF_{\tilde{x}+\widetilde{\Delta x}-\pi(\bar{\alpha})}(\omega)]}{\int U_1(\omega;\bar{F}(\cdot,\bar{\alpha}))[(1-\bar{\alpha})\,dF_{\tilde{x}+\widetilde{\Delta x}+\tilde{\epsilon}-\pi(\bar{\alpha})}(\omega)+\bar{\alpha}\,dF_{\tilde{x}+\widetilde{\Delta x}-\pi(\bar{\alpha})}(\omega)]}-\pi'(\bar{\alpha}).$$

Substituting in (10), recalling that $E[\tilde{\epsilon}\,|\,x]\equiv E[\tilde{\epsilon}\,|\,x+\Delta x]\equiv 0$, and applying tedious algebra yields that (11) is equal to

$$\int\!\!\int\Bigg[\int_0^\infty\!\int_0^\epsilon\!\int_0^s\Bigg\{\frac{-U_{11}(x+\Delta x-\pi(\bar{\alpha})+r;\bar{F}(\cdot,\bar{\alpha}))}{\int U_1(\omega;\bar{F}(\cdot,\bar{\alpha}))\,d\bar{F}(\omega,\bar{\alpha})}$$

$$-\frac{-U_{11}(x-\pi(\bar{\alpha})+r;F(\cdot,\bar{\alpha}))}{\int U_1(\omega;F(\cdot,\bar{\alpha}))\,dF(\omega,\bar{\alpha})}\Bigg\}\,dr\cdot ds\cdot dF_{\tilde{\epsilon}\,|\,x,\Delta x}(\epsilon\,|\,x,\Delta x)$$

$$+\int_{-\infty}^0\!\int_\epsilon^0\!\int_s^0\Bigg\{\frac{-U_{11}(x+\Delta x-\pi(\bar{\alpha})+r;\bar{F}(\cdot,\bar{\alpha}))}{\int U_1(\omega;\bar{F}(\cdot,\bar{\alpha}))\,d\bar{F}(\omega,\bar{\alpha})}$$

$$-\frac{-U_{11}(x-\pi(\bar{\alpha})+r;F(\cdot\,\bar{\alpha}))}{\int U_1(\omega;F(\cdot,\bar{\alpha}))\,dF(\omega,\bar{\alpha})}\Bigg\}dr$$

$$\cdot\, ds\cdot dF_{\tilde{\epsilon}\,|\,x,\Delta x}(\epsilon\,|\,x,\Delta x)\Bigg]\,dF_{\tilde{x},\widetilde{\Delta x}}(x,\Delta x).$$

Since $\widetilde{\Delta x}\geq 0$ and $\bar{F}(\cdot,\bar{\alpha})$ equals or stochastically dominates $F(\cdot,\bar{\alpha})$ for all $\bar{\alpha}$, it follows that $dV(\bar{F}(\cdot,\alpha))/d\alpha|_{\bar{\alpha}}\leq 0$ for all $\bar{\alpha}$, so that integrating this derivative from $\bar{\alpha}=0$ to $\bar{\alpha}=1$ yields $0\geq V(\bar{F}(\cdot,1))-V(\bar{F}(\cdot,0))=V(F_{\tilde{x}+\widetilde{\Delta x}-\pi_0})-V(F_{\tilde{x}+\widetilde{\Delta x}+\tilde{\epsilon}})$, which by monotonicity (see footnote 11) yields $\pi_1\leq\pi_0$.

(ii)→(i): Assume $-U_{11}(y;F_{\tilde{z}})/\int U_1(\omega;F_{\tilde{z}})\,dF_{\tilde{z}}(\omega)<2\beta<-U_{11}(y^*;F_{\tilde{z}^*})/\int U_1(\omega;F_{\tilde{z}^*})\,dF_{\tilde{z}^*}(\omega)$ where $y\leq y^*$ and $F_{\tilde{z}^*}(\cdot)$ equals or stochastically dominates $F_{\tilde{z}}(\cdot)$. Applying equation (4) and simplifying yields

$$\frac{d}{dp}\left[V\left((1-p)F_{\tilde{z}}+\frac{p}{2}G_{y+\sqrt{e}}+\frac{p}{2}G_{y-\sqrt{e}}\right)-V((1-p)F_{\tilde{z}-pe\beta}+pG_{y-pe\beta})\right]\Bigg|_{p=0}$$

$$=\tfrac{1}{2}U(y+\sqrt{e}\,;F_{\tilde{z}})+\tfrac{1}{2}U(y-\sqrt{e}\,;F_{\tilde{z}})-U(y;F_{\tilde{z}})+e\beta\cdot\int U_1(\omega;F_{\tilde{z}})\,dF_{\tilde{z}}(\omega),$$

which, since $\lim_{e\to 0}(1/e)[\frac{1}{2}U(y+\sqrt{e}\,;F_{\tilde{z}})+\frac{1}{2}U(y-\sqrt{e}\,;F_{\tilde{z}})-U(y;F_{\tilde{z}})]=\frac{1}{2}U_{11}(y;F_{\tilde{z}})$, is positive for small enough positive e. Replacing $(y,F_{\tilde{z}})$ with $(y^*,F_{\tilde{z}^*})$ and repeating yields that for some $p,e>0$,

$$(12)\qquad V\left((1-p)F_{\tilde{z}}+\frac{p}{2}G_{y+\sqrt{e}}+\frac{p}{2}G_{y-\sqrt{e}}\right)>V((1-p)F_{\tilde{z}-pe\beta}+pG_{y-pe\beta}),\qquad\text{and}$$

$$(13)\qquad V\left((1-p)F_{\tilde{z}^*}+\frac{p}{2}G_{y^*+\sqrt{e}}+\frac{p}{2}G_{y^*-\sqrt{e}}\right)<V((1-p)F_{\tilde{z}^*-pe\beta}+pG_{y^*-pe\beta}).$$

By Theorem 1 of Hansen, Holt, and Peled [4] there exists a distribution $F_{\tilde{r},\tilde{s}}(\cdot,\cdot)$ with $\tilde{r},\tilde{s}\geq 0$ for which $F_{\tilde{r}}(\cdot)=F_{\tilde{z}}(\cdot)$ and $F_{\tilde{r}+\tilde{s}}(\cdot)=F_{\tilde{z}^*}(\cdot)$. Let $\text{prob}((\tilde{r},\tilde{s})=(y,y^*-y))=\gamma$. Defining $F_{\tilde{x},\widetilde{\Delta x}}(\cdot,\cdot)$

$= (1-p)F_{\tilde{r},\tilde{s}}(\cdot,\cdot) + pG_{y,y^*-y}(\cdot,\cdot)$ and

$$F_{\tilde{\epsilon}\,|\,x,\Delta x}(\cdot\,|\,x,\Delta x) = \begin{cases} \dfrac{\gamma - p\gamma}{p+\gamma-p\gamma}G_0(\cdot) + \dfrac{\frac{1}{2}p}{p+\gamma-p\gamma}G_{\sqrt{e}}(\cdot) + \dfrac{\frac{1}{2}p}{p+\gamma-p\gamma}G_{-\sqrt{e}}(\cdot) & \\ \qquad\qquad\qquad\qquad\qquad\qquad\qquad \text{if } (x,\Delta x) = (y, y^*-y), & \\ G_0(\cdot) \quad \text{otherwise,} & \end{cases}$$

we have that the distribution functions of $\tilde{x}+\tilde{\epsilon}$, $\tilde{x}-pe\beta$, $\tilde{x}+\widetilde{\Delta x}+\tilde{\epsilon}$, and $\tilde{x}+\widetilde{\Delta x}-pe\beta$ are given by the four respective arguments of $V(\cdot)$ in (12) and (13). This implies $\pi_0 < pe\beta < \pi_1$, which is a contradiction.

(i)$\to$(iii): If for some $(\tilde{x}, \widetilde{\Delta x}, \epsilon)$ and $r \geq 0$ we had $\alpha_1 < \alpha_0$, then since the individual is a diversifier there will exist some positive $\alpha^* \in (\alpha_1, \alpha_0)$ such that

$$0 < \frac{d}{d\alpha}\left(V(F_{\tilde{x}+\alpha r+\alpha\tilde{\epsilon}})\right)\Big|_{\alpha^*}$$

$$= \int_0^M\int_0^M\int_{-\infty}^{+\infty}(r+\epsilon)\cdot U_1(x+\alpha^*r+\alpha^*\epsilon; F_0)\,dF_{\tilde{x},\widetilde{\Delta x},\tilde{\epsilon}}(x,\Delta x,\epsilon),$$

where $F_0(\cdot) = F_{\tilde{x}+\alpha^*r+\alpha^*\tilde{\epsilon}}(\cdot)$. Since $U_1(\cdot, F) > 0$, this implies

$$r > -\left[\int_0^M\int_0^M\int_{-\infty}^{+\infty}\epsilon\cdot U_1(x+\alpha^*r+\alpha^*\epsilon; F_0)\,dF_{\tilde{x},\widetilde{\Delta x},\tilde{\epsilon}}(x,\Delta x,\epsilon)\right]\Big/$$

$$\left[\int_0^M U_1(\omega; F_0)\,dF_0(\omega)\right]$$

$$= \int_0^M\int_0^M\left[\int_0^\infty \epsilon\int_0^{\alpha^*\epsilon}\left\{\frac{-U_{11}(x+\alpha^*r+s; F_0)}{\int U_1(\omega; F_0)\,dF_0(\omega)}\right\}ds\cdot dF_{\tilde{\epsilon}\,|\,x,\Delta x}(\epsilon\,|\,x,\Delta x)\right.$$

$$+\int_{-\infty}^0(-\epsilon)\int_{\alpha^*\epsilon}^0\left\{\frac{-U_{11}(x+\alpha^*r+s; F_0)}{\int U_1(\omega; F_0)\,dF_0(\omega)}\right\}ds$$

$$\left.\cdot\, dF_{\tilde{\epsilon}\,|\,x,\Delta x}(\epsilon\,|\,x,\Delta x)\right]dF_{\tilde{x},\widetilde{\Delta x}}(x,\Delta x).$$

By (i), this implies

$$r > \int_0^M\int_0^M\left[\int_0^\infty \epsilon\int_0^{\alpha^*\epsilon}\left\{\frac{-U_{11}(x+\Delta x+\alpha^*r+s; F_1)}{\int U_1(\omega; F_1)\,dF_1(\omega)}\right\}ds\cdot dF_{\tilde{\epsilon}\,|\,x,\Delta x}(\epsilon\,|\,x,\Delta x)\right.$$

$$+\int_{-\infty}^0(-\epsilon)\int_{\alpha^*\epsilon}^0\left\{\frac{-U_{11}(x+\Delta x+\alpha^*r+s; F_1)}{\int U_1(\omega; F_1)\,dF_1(\omega)}\right\}ds$$

$$\left.\cdot\, dF_{\tilde{\epsilon}\,|\,x,\Delta x}(\epsilon\,|\,x,\Delta x)\right]dF_{\tilde{x},\widetilde{\Delta x}}(x,\Delta x)$$

$$= -\left[\int_0^M\int_0^M\int_{-\infty}^{+\infty}\epsilon\cdot U_1(x+\Delta x+\alpha^*r+\alpha^*\epsilon; F_1)\,dF_{\tilde{x},\widetilde{\Delta x},\tilde{\epsilon}}(x,\Delta x,\epsilon)\right]\Big/$$

$$\left[\int_0^M U_1(\omega; F_1)\,dF_1(\omega)\right],$$

where $F_1(\cdot) = F_{\tilde{x}+\widetilde{\Delta x}+\alpha^* r+\alpha^*\tilde{\epsilon}}(\cdot)$. This then implies

$$0 < \int_0^M \int_0^M \int_{-\infty}^{+\infty} (r+\epsilon)\cdot U_1(x+\Delta x+\alpha^* r+\alpha^*\epsilon; F_1)\, dF_{\tilde{x},\widetilde{\Delta x},\tilde{\epsilon}}(x,\Delta x,\epsilon)$$

$$= \frac{d}{d\alpha}\left(V(F_{\tilde{x}+\widetilde{\Delta x}+\alpha r+\alpha\tilde{\epsilon}})\right)\Big|_{\alpha^*},$$

which, since $\alpha_1 < \alpha^*$, contradicts the assumption that the individual is a diversifier.

(iii)→(i): Assume $-U_{11}(y;F_{\tilde{w}})/\int U_1(\xi;F_{\tilde{w}})\,dF_{\tilde{w}}(\xi) < -U_{11}(y^*;F_{\widetilde{w^*}})/\int U_1(\xi;F_{\widetilde{w^*}})\,dF_{\widetilde{w^*}}(\xi)$, where $y \le y^*$ and $F_{\widetilde{w^*}}(\cdot)$ equals or stochastically dominates $F_{\tilde{w}}(\cdot)$. Since $\lim_{\alpha\to 0}\frac{1}{2}(U_1(y+\alpha;F_{\tilde{w}}) - U_1(y-\alpha;F_{\tilde{w}}))/\alpha = U_{11}(y;F_{\tilde{w}})$ and similarly for $y^*, F_{\widetilde{w^*}}(\cdot)$, we have that for some small positive $\bar{\alpha}$ and positive β that

$$\frac{-\frac{1}{2}(U_1(y+\bar{\alpha};F_{\tilde{w}}) - U_1(y-\bar{\alpha};F_{\tilde{w}}))}{\int U_1(\xi;F_{\tilde{w}})\,dF_{\tilde{w}}(\xi)} < \beta < \frac{-\frac{1}{2}(U_1(y^*+\bar{\alpha};F_{\widetilde{w^*}}) - U_1(y^*-\bar{\alpha};F_{\widetilde{w^*}}))}{\int U_1(\xi;F_{\widetilde{w^*}})\,dF_{\widetilde{w^*}}(\xi)}.$$

Define

$$\phi(\alpha,p) \equiv V\big((1-p)F_{\tilde{w}+p\beta(\alpha-\bar{\alpha})} + \tfrac{1}{2}pG_{y+(p\beta+1)\alpha} + \tfrac{1}{2}pG_{y+(p\beta-1)\alpha}\big)$$

and

$$\phi^*(\alpha,p) \equiv V\big((1-p)F_{\widetilde{w^*}+p\beta(\alpha-\bar{\alpha})} + \tfrac{1}{2}pG_{y^*+(p\beta+1)\alpha} + \tfrac{1}{2}pG_{y^*+(p\beta-1)\alpha}\big).$$

Applying (4) and simplifying yields

$$\phi_{\alpha p}(\bar{\alpha},0) = \beta\int U_1(\xi;F_{\tilde{w}})\,dF_{\tilde{w}}(\xi) + \tfrac{1}{2}(U_1(y+\bar{\alpha};F_{\tilde{w}}) - U_1(y-\bar{\alpha};F_{\tilde{w}})) > 0,$$

and

$$\phi^*_{\alpha p}(\bar{\alpha},0) = \beta\int U_1(\xi,F_{\widetilde{w^*}})\,dF_{\widetilde{w^*}}(\xi) + \tfrac{1}{2}(U_1(y^*+\bar{\alpha};F_{\widetilde{w^*}}) - U_1(y^*-\bar{\alpha};F_{\widetilde{w^*}})) < 0,$$

which, since $\phi_\alpha(\bar{\alpha},0) = \phi^*_\alpha(\bar{\alpha},0) = 0$, implies that $\phi_\alpha(\bar{\alpha},\bar{p}) > 0$ and $\phi^*_\alpha(\bar{\alpha},\bar{p}) < 0$ for some small positive $\bar{p}$.

Since $F_{\widetilde{w^*}-\bar{p}\beta\bar{\alpha}}(\cdot)$ stochastically dominates $F_{\tilde{w}-\bar{p}\beta\bar{\alpha}}(\cdot)$, we have by Theorem 1 of Hansen, Holt, and Peled [4] that there exists a bivariate distribution $F_{\tilde{r},\tilde{s}}(\cdot,\cdot)$ with $\tilde{r},\tilde{s} \ge 0$ and such that

$$F_{\tilde{r}}(\cdot) = F_{\tilde{w}-\bar{p}\beta\bar{\alpha}}(\cdot) \quad \text{and} \quad F_{\tilde{r}+\tilde{s}}(\cdot) = F_{\widetilde{w^*}-\bar{p}\beta\bar{\alpha}}(\cdot).$$

Let

$$\text{prob}((\tilde{r},\tilde{s}) = (y, y^*-y)) = \gamma,$$

let

$$F_{\tilde{x},\widetilde{\Delta x}}(\cdot,\cdot) = (1-\bar{p})F_{\tilde{r},\tilde{s}}(\cdot,\cdot) + \bar{p}G_{y,y^*-y}(\cdot,\cdot),$$

let

$$F_{\tilde{\epsilon}\mid x,\Delta x}(\cdot\mid x,\Delta x) = \begin{cases} \dfrac{\gamma-\bar{p}\gamma}{\bar{p}+\gamma-\bar{p}\gamma}G_0(\cdot) + \dfrac{\frac{1}{2}\bar{p}}{\bar{p}+\gamma-\bar{p}\gamma}G_1(\cdot) + \dfrac{\frac{1}{2}\bar{p}}{\bar{p}+\gamma-\bar{p}\gamma}G_{-1}(\cdot) & \text{if } (x,\Delta x) = (y, y^*-y), \\ G_0(\cdot) \quad \text{otherwise,} \end{cases}$$

and let $r = \bar{p}\beta$. Then for any α,

$$V(F_{\tilde{x}+\alpha r+\alpha\tilde{\epsilon}}) = V\big((1-\bar{p})F_{\tilde{w}+\bar{p}\beta(\alpha-\bar{\alpha})} + \tfrac{1}{2}\bar{p}G_{y+(\bar{p}\beta+1)\alpha} + \tfrac{1}{2}\bar{p}G_{y+(\bar{p}\beta-1)\alpha}\big)$$
$$= \phi(\alpha, \bar{p})$$

and

$$V(F_{\tilde{x}+\widetilde{\Delta x}+\alpha r+\alpha\tilde{\epsilon}}) = V\big((1-\bar{p})F_{\widetilde{w^*}+\bar{p}\beta(\alpha-\bar{\alpha})} + \tfrac{1}{2}\bar{p}G_{y^*+(\bar{p}\beta+1)\alpha} + \tfrac{1}{2}\bar{p}G_{y^*+(\bar{p}\beta-1)\alpha}\big)$$
$$= \phi^*(\alpha, \bar{p}).$$

Since the individual is a diversifier and $\phi_\alpha(\bar{\alpha}, \bar{p}) > 0 > \phi^*_\alpha(\bar{\alpha}, \bar{p})$, we have that the value of α which maximizes $V(F_{\tilde{x}+\alpha r+\alpha\tilde{\epsilon}})$ is greater than $\bar{\alpha}$ and the value which maximizes $V(F_{\tilde{x}+\widetilde{\Delta x}+\alpha r+\alpha\tilde{\epsilon}})$ is less than $\bar{\alpha}$, contradicting (iii). *Q.E.D.*

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